

Answer all of the following questions. Each question is worth 10 marks.

Calculators are NOT allowed

1. (a) Solve the inequality: $\frac{x-1}{x-3} < \frac{x-3}{1-x}$.

(b) Find the domain of f where:

$$f(x) = \frac{\sqrt{x^2 - 9}}{(x-4)\sqrt{25-x^2}}$$

2. (a) Find the equation of the line perpendicular to the line $y - 2x + 3 = 0$ and passing through the center of the circle $x^2 + y^2 - 2x + 4y + 3 = 0$.

(b) Prove the trigonometric identity: $\csc^2\theta - \sec^2\theta = 4 \cot 2\theta \csc 2\theta$

3. (a) Use the definition of the limit to show that:

$$\lim_{x \rightarrow -2} (3x - 7) = -13$$

(b) Use the sandwich theorem to evaluate $\lim_{x \rightarrow 1} \frac{(x-1)^2}{\sqrt{x^4 + 2x^2 + 8}}$.

4. (a) Let :

$$f(x) = \begin{cases} A[x] & , \text{ if } x < -1 \\ \frac{x^2 - x - 2}{x + 1} & , \text{ if } x > -1 \end{cases}$$

Find A so that $\lim_{x \rightarrow -1} f(x)$ exists.

(b) Find all values of θ in $[0, 2\pi)$ which satisfy : $2 \sin^2\theta + \cos\theta - 1 = 0$.

Good Luck